ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Find the values of A, B and C in the identity $4x^2 - 16x + C \equiv A(x + B)^2 + 2$. **[4]**

2 You are given that $\mathbf{M} = \begin{pmatrix} 2 & -5 \\ 3 & 7 \end{pmatrix}$.

$\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \end{pmatrix}$ represents two simultaneous equations.

(i) Write down these two equations. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and use it to solve the equations. **[4]**

3 The cubic equation $2z^3 - z^2 + 4z + k = 0$, where k is real, has a root $z = 1 + 2\text{j}$.

Write down the other complex root. Hence find the real root and the value of k. **[6]**

4 The roots of the cubic equation $x^3 - 2x^2 - 8x + 11 = 0$ are α, β and γ.

Find the cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$. **[6]**

5 Use the result $\dfrac{1}{5r-1} - \dfrac{1}{5r+4} \equiv \dfrac{5}{(5r-1)(5r+4)}$ and the method of differences to find

$$\sum_{r=1}^{n} \frac{1}{(5r-1)(5r+4)},$$

simplifying your answer. **[6]**

6 A sequence is defined by $u_1 = 2$ and $u_{n+1} = \dfrac{u_n}{1 + u_n}$.

(i) Calculate u_3. **[2]**

(ii) Prove by induction that $u_n = \dfrac{2}{2n-1}$. **[6]**

Section B (36 marks)

7 Fig. 7 shows an incomplete sketch of $y = \dfrac{(2x-1)(x+3)}{(x-3)(x-2)}$.

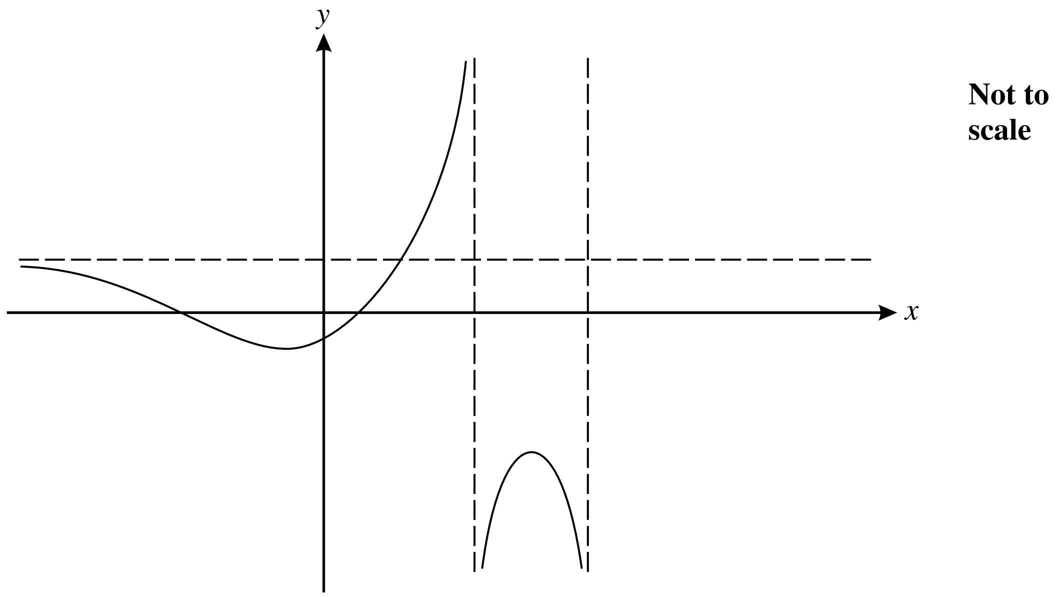

Fig. 7

(i) Find the coordinates of the points where the curve cuts the axes. **[2]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for large positive values of x, justifying your answer. Copy and complete the sketch. **[3]**

(iv) Solve the inequality $\dfrac{(2x-1)(x+3)}{(x-3)(x-2)} < 2$. **[4]**

8 Two complex numbers, α and β, are given by $\alpha = \sqrt{3} + \mathrm{j}$ and $\beta = 3\mathrm{j}$.

(i) Find the modulus and argument of α and β. **[3]**

(ii) Find $\alpha\beta$ and $\dfrac{\beta}{\alpha}$, giving your answers in the form $a + b\mathrm{j}$, showing your working. **[5]**

(iii) Plot α, β, $\alpha\beta$ and $\dfrac{\beta}{\alpha}$ on a single Argand diagram. **[2]**

[Question 9 is printed overleaf.]

9 The matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ represent transformations P and Q respectively.

(i) Describe fully the transformations P and Q. **[4]**

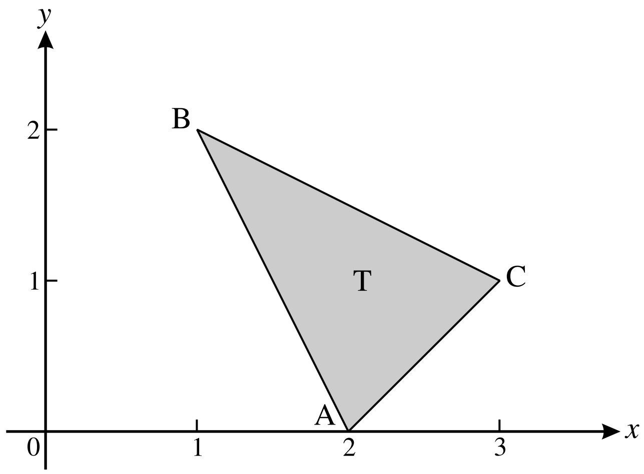

Fig. 9

Fig. 9 shows triangle T with vertices A (2, 0), B (1, 2) and C (3, 1).

Triangle T is transformed first by transformation P, then by transformation Q.

(ii) Find the single matrix that represents this composite transformation. **[2]**

(iii) This composite transformation maps triangle T onto triangle T′, with vertices A′, B′ and C′. Calculate the coordinates of A′, B′ and C′. **[2]**

T′ is reflected in the line $y = -x$ to give a new triangle, T″.

(iv) Find the matrix **R** that represents reflection in the line $y = -x$. **[2]**

(v) A single transformation maps T″ onto the original triangle, T. Find the matrix representing this transformation. **[4]**